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A Hybrid Problem in Plane Elasticity

Surendra K. Dhir

COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT
DEPARTMENTAL REPORT

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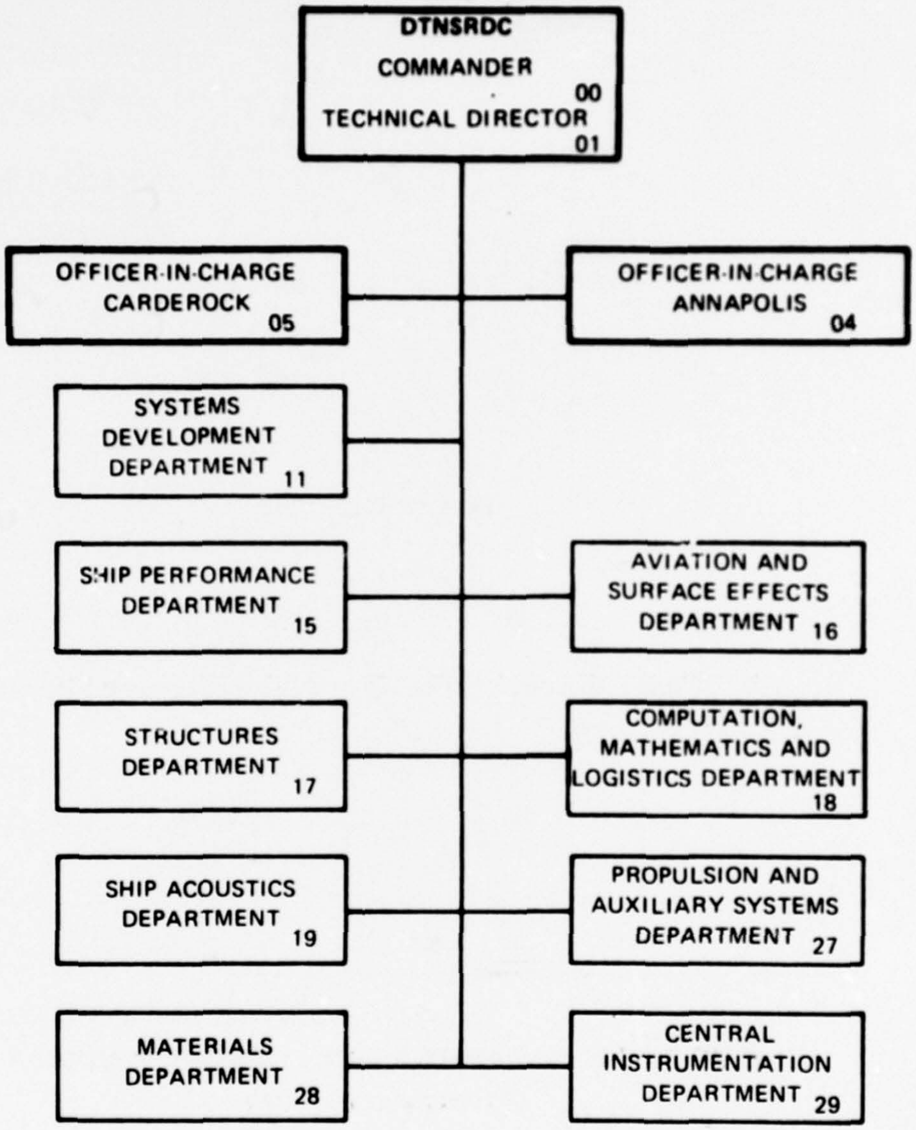

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A fundamental problem in plane elasticity is studied: to determine for a given opening the amount and distribution of reinforcing material required to satisfy certain specified stress field criteria. The relationship between the local stress field around the opening and the reinforcement distribution is presented explicitly such that either one of them can assume the role of an independent parameter. The problem is set up as Muskhelishvili's first boundary value problem in which a function is derived to represent the loads

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NOTATION

A	Cross-sectional area of reinforcement
a_n	Coefficients of mapping function
a'_n	Coefficients of complex function $\phi(\zeta)$
b_n	Coefficients of complex function $\psi(\zeta)$
c	Constant
c_n, d_n	Coefficients of P function
D	Related to difference of principal stresses
ds	Element of arc length
$F(\sigma)$	Function related to radius of curvature
i	$\sqrt{-1}$
m,n	Running indices
P	Axial force in reinforcement
p,q	Principal stresses at infinity
R	Size of opening
R^*	Normal force
r	Radial coordinate (polar)
S	Related to sum of principal stresses
T	Tangential force
t	Thickness of plate
U	Airy stress function
x,y	Cartesian coordinates
z	Complex number, $x + iy$
α, β	Curvilinear coordinates
θ	Polar angle
$\sigma_\alpha, \sigma_\beta, \tau_{\alpha\beta}$	Curvilinear stresses
$\phi(\ ), \psi(\ )$	Complex functions of argument
σ	Boundary value of ζ at $e^\alpha = \rho = 1$
ζ	$e^{\alpha + i\beta}$
ν	Poisson's ratio

ABSTRACT

A fundamental problem in plane elasticity is studied: to determine for a given opening the amount and distribution of reinforcing material required to satisfy certain specified stress field criteria. The relationship between the local stress field around the opening and the reinforcement distribution is presented explicitly such that either one of them can assume the role of an independent parameter. The problem is set up as Muskhelishvili's first boundary value problem in which a function is derived to represent the loads due to reinforcement. A solution is worked out for a general mapping function which can describe almost any opening of practical interest. Numerical results and other implications for a reinforced circular opening are discussed in greater detail.

INTRODUCTION

The use of reinforcements at the boundaries of openings of various shapes is common practice in ship and aircraft construction. Beginning with the Kirsch solution of a circular opening in an infinite plate under uniform tension, there are in the literature a great many papers which deal with the boundary value problem of the reinforcement of openings in plates. With a few notable exceptions [1,2] most researchers formulate these investigations as a stress/strain boundary value problem, i.e., the stresses and/or strains are specified (constrained) at infinity as well as at hole boundaries and the resulting stress field is then computed. The approach is therefore one of trial-and-error in which several designs of reinforced openings must be studied before acceptable stress levels are found.

Mansfield [1] attempted the inverse approach of specifying the complete stress field in the plate and then seeking a compatible opening with variable reinforcement. His stress criteria required the original stress field to be unperturbed by the presence of reinforced holes. Mansfield, therefore, used the phrase "neutral holes". The results he produced were rather limited in application, as some of the reinforcements required to render the holes neutral were not very practicable. Recently, Bjorkman and Richards [2] studied the inverse plane elasticity problem of an infinite plate containing a hole which leaves the first stress invariant ($\sigma_x + \sigma_y$) of the original stress field unchanged. They called such holes "harmonic holes"; the phrase is justified in that $\sigma_x + \sigma_y$ is a harmonic function which remains unaffected by the presence of such a hole. They have used the usual Muskhelishvili formalism and have further shown that an ellipse is a possible shape for a harmonic hole where the ratio of major to minor axes bears a direct relationship to the ratio of the normal stresses at infinity in two directions. While the treatment of the problem by Bjorkman and Richards [2] is quite elegant, the actual application is again somewhat limited due to the shape (elliptical) and unreinforced nature of the holes.

The approach followed in the present paper builds partly upon an earlier paper by the author [3] and partly upon the "harmonic hole" condition. The problem can thus be formulated: determine the distribution of reinforcement around a specified opening in an infinite plate under bi-axial loading such that the sum of the principal stresses (or the first invariant of the stress tensor) remains unchanged.

BOUNDARY CONDITIONS

Figure 1 shows an opening of general shape which is reinforced with a variable amount of reinforcement. The forces acting on a boundary element, ds, of the reinforcement are shown in Figures 2a and 2b. If this boundary is to be in elastic equilibrium, the following equations must hold true (in curvilinear coordinates, Figure 2a):

$$\frac{\partial P}{\partial s} ds + T\,ds = 0, \qquad R^* ds - P\,d\beta = 0,$$
$$R^* = \sigma_\alpha t, \qquad T = \tau_{\alpha\beta} t; \tag{1A}$$

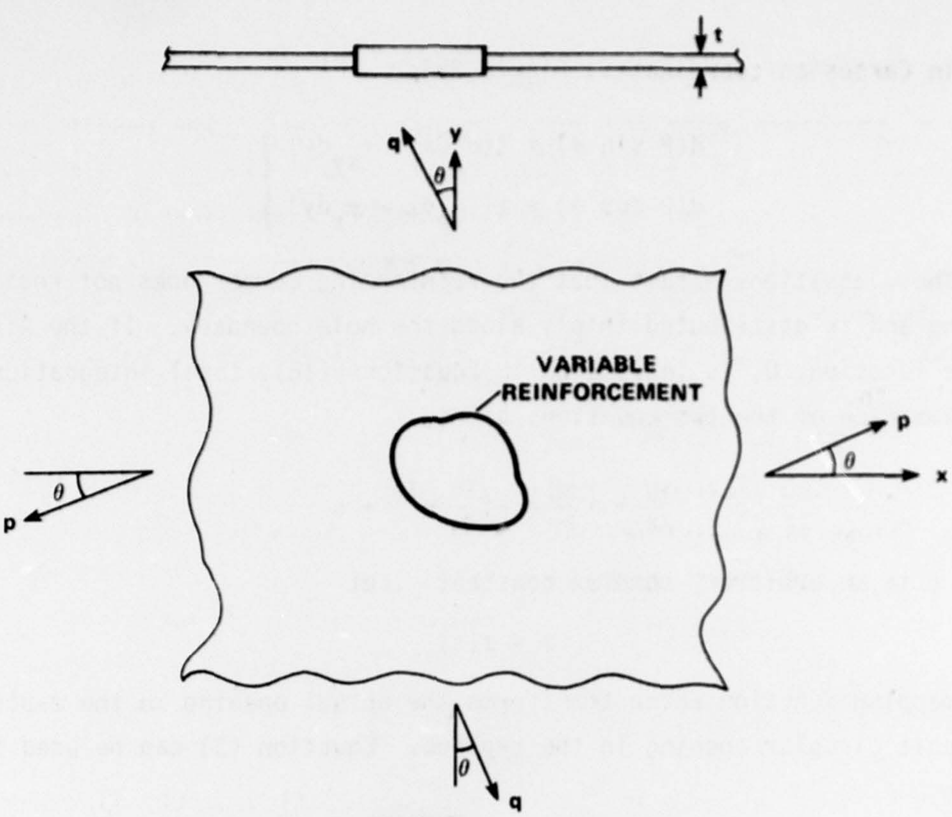

Figure 1 - A Reinforced Hole in a Plate

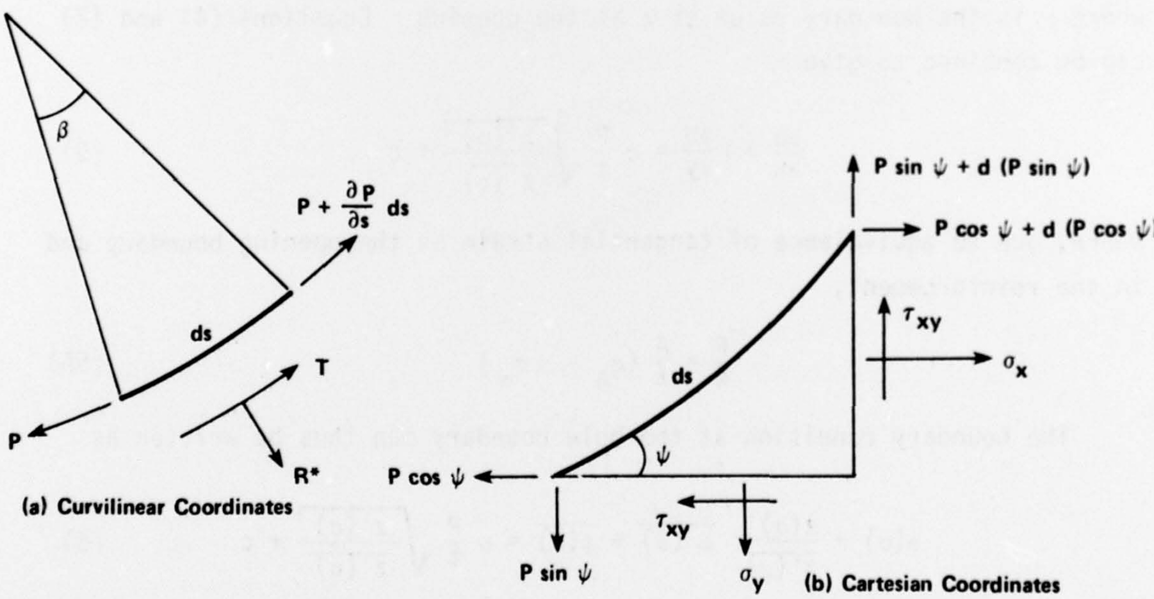

Figure 2 - Forces on an Element of Reinforcement

and (in Cartesian coordinates, Figure 2b):

$$\left.\begin{aligned} d(P \sin \psi) &= t(\sigma_y dx - \tau_{xy} dy) \\ d(P \cos \psi) &= t(\tau_{xy} dx - \sigma_x dy) \end{aligned}\right\} \qquad (1B)$$

These equations assume that the reinforcing member does not resist bending and is distributed thinly along the hole boundary. If the Airy stress function, U, is introduced in Equations (1B), total integration and summation of the two equations gives

$$\frac{\partial U}{\partial x} + i\frac{\partial U}{\partial y} = -i\frac{P}{t}\, e^{i\psi} + c \qquad (2)$$

where c is an arbitrary complex constant. Let

$$z = z(\zeta) \qquad (3)$$

be a mapping function which transforms the actual opening in the z-plane to a unit circular opening in the ζ-plane. Equation (3) can be used to obtain

$$e^{i\psi} = i\,\sigma\sqrt{\frac{\overline{z'(\sigma)}}{z'(\sigma)}} \qquad (4)$$

where σ is the boundary value of ζ at the opening. Equations (4) and (2) can be combined to give

$$\frac{\partial U}{\partial x} + i\frac{\partial U}{\partial y} = \sigma\,\frac{P}{t}\sqrt{\frac{\overline{z'(\sigma)}}{z'(\sigma)}} + c \qquad (5)$$

where, due to equivalence of tangential strain at the opening boundary and in the reinforcement,

$$\frac{P}{t} = \frac{A}{t}\,(\sigma_\beta - \nu\,\sigma_\alpha) \qquad (5A)$$

The boundary condition at the hole boundary can thus be written as

$$\phi(\sigma) + \frac{z(\sigma)}{z'(\sigma)}\;\overline{\phi'(\sigma)} + \overline{\psi(\sigma)} = \sigma\,\frac{P}{t}\sqrt{\frac{\overline{z'(\sigma)}}{z'(\sigma)}} + c \qquad (6)$$

where $\phi(\sigma)$ and $\psi(\sigma)$ are boundary values of the functions $\phi(\zeta)$ and $\psi(\zeta)$ at the opening. These two complex functions define the entire stress field by

$$\left.\begin{aligned} &\sigma_\alpha + \sigma_\beta = 2\left[\frac{\phi'(\zeta)}{z'(\zeta)} + \overline{\frac{\phi'(\zeta)}{z'(\zeta)}}\right] \\ \text{and} \\ &\sigma_\beta - \sigma_\alpha + 2\,i\,\tau_{\alpha\beta} = \frac{2\zeta^2}{\rho^2\,\overline{z'(\zeta)}}\left[\overline{z(\zeta)}\,\Phi'(\zeta) + \psi'(\zeta)\right] \end{aligned}\right\} \qquad (7)$$

where

$$\Phi(\zeta) = \frac{\phi'(\zeta)}{z'(\zeta)}$$

In order that Equations (7) also predict the stresses at infinity, the complex functions $\phi(\zeta)$ and $\psi(\zeta)$ assume the following forms

$$\left.\begin{aligned} \phi(\zeta) &= S\,z(\zeta) + \phi_0(\zeta) \\ \psi(\zeta) &= D\,z(\zeta) + \psi_0(\zeta) \end{aligned}\right\} \qquad (8)$$

where S and D are constants and the functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$ approach zero at large values of ζ. Thus

$$\left.\begin{aligned} \phi_0(\zeta) &= \sum_{n=1}^{\infty} \frac{a'_n}{\zeta^n} \\ \psi_0(\zeta) &= \sum_{n=0}^{\infty} \frac{b_n}{\zeta^n} \end{aligned}\right\} \qquad (9)$$

The constants S and D can easily be computed, using Equations (7), to be

$$\left.\begin{aligned} S &= \frac{p + q}{4} \\ D &= -\frac{p - q}{2}\,e^{-2\,i\theta} \end{aligned}\right\} \qquad (10)$$

where p and q are the stresses at infinity and θ is the angle between the x-axis and p. The form of $\phi(\zeta)$ and $\psi(\zeta)$ can, thus, be effectively used to automatically satisfy the boundary condition at infinity and Equation (6) would then be the only remaining condition to be satisfied for obtaining a solution to the problem. It should be noted that Equation (6) is considerably simpler than a comparable equation developed earlier by the author [3] for solving the direct problems of elasticity involving reinforced hole boundaries.

METHOD OF SOLUTION

Once the boundary condition as given by Equation (6) is developed, its solution is merely routine and may or may not be complicated depending on the complexity of the mapping function. In this case, however, this equation is further simplified by specifying a field condition which requires

$$\sigma_\alpha + \sigma_\beta = 2\left[\frac{\phi'(\zeta)}{z'(\zeta)} + \frac{\overline{\phi'(\zeta)}}{\overline{z'(\zeta)}}\right] = p + q \tag{11}$$

Equation (11) reduces to $\phi_0(\zeta) = 0$,* as shown by Bjorkman and Richards [2]. Thus Equation (6) becomes

$$2S\,z(\sigma) + D\,\overline{z(\sigma)} + \overline{\psi_0(\sigma)} = \sigma\,\frac{P}{t}\sqrt{\frac{\overline{z'(\sigma)}}{z'(\sigma)}} + c \tag{12}$$

If the opening is assumed to be a smooth closed curve, it is possible to represent

$$\frac{P}{t}\sqrt{z'(\sigma)\,\overline{z'(\sigma)}} = \sum_{n=0}^{\infty}\left(c_n\sigma^n + \frac{d_n}{\sigma^n}\right) \tag{13}$$

Since $\frac{P}{t}\sqrt{z'(\sigma)\,\overline{z'(\sigma)}}$ is a real quantity, c_n and d_n have to be accordingly adjusted. Equations (12) and (13) can be combined to obtain

$$2S\,z(\sigma)\,\overline{z'(\sigma)} + \overline{D}\,\overline{z(\sigma)}\,\overline{z'(\sigma)} + \overline{\psi_0(\sigma)}\,\overline{z'(\sigma)} =$$

$$\sum_{n=0}^{\infty}\left(c_n\sigma^{n+1} + \frac{d_n}{\sigma^{n-1}}\right) + c\,\overline{z'(\sigma)} \tag{14}$$

which is now the boundary condition at the reinforced opening of any general shape. It is easy to show that, with Muskhelishvili's technique,

---

* Care must be taken since Bjorkman and Richards [2] use different notation.

Equation (14) can conveniently be solved for a general mapping function of the type

$$z(\zeta) = R(\zeta + \sum_{m=1}^{\infty} \frac{a_m}{\zeta^m}) \qquad (15)$$

which can represent almost any opening of practical interest. However, to focus attention on a particular opening and to study the related implications, the problem of a circular opening will now be discussed in greater detail.

CIRCULAR OPENING

Let a plane plate under biaxial uniform tensions p and q in the x- and y-directions, respectively (see Equations (10)), contain a circular opening of radius R. The mapping function

$$z = z(\zeta) = R\zeta \; ; \quad \zeta = r\, e^{i\theta} \qquad (16)$$

which describes a circular geometry, can be substituted in the boundary condition (Equation (14)) to obtain

$$2S\, R^2\sigma + \frac{DR^2}{\sigma} + R \sum_{n=0}^{\infty} \bar{b}_n \sigma^n = \sum_{n=0}^{\infty} (c_n \sigma^{n+1} + \frac{d_n}{\sigma^{n-1}}) + cR \qquad (17)$$

Now Equation (17) and its conjugate can be multiplied by $\frac{1}{2\pi i} \frac{d\sigma}{\sigma-\zeta}$ and integrated around the unit circle to obtain

$$\begin{array}{llll} c_n = d_n; & n \geq 0 & b_0 = c = 0 & \\ 2c_0 = b_1 R + 2S\, R^2 & & b_2 = 0 & \\ c_1 = 0 & & b_3 = D\, R & (18) \\ c_2 = D\, R^2 & & b_n = 0; & n \geq 4 \\ c_n = 0; & n \geq 3 & & \end{array}$$

which define the solution to the problem. The functions $\phi(\zeta)$, $\psi(\zeta)$, and $\frac{P}{t}$ can now be given by

$$\left.\begin{aligned}\phi(\zeta) &= SR\,\zeta \\ \psi(\zeta) &= DR\,\zeta + \frac{b_1}{\zeta} + \frac{DR}{\zeta^3} \\ \frac{p}{t} &= b_1 + 2SR + DR(\sigma^2 + \frac{1}{\sigma^2})\end{aligned}\right\} \qquad (19)$$

Equations (5A), (7), and (19) can be combined to obtain

$$\left.\begin{aligned}\sigma_r &= 2S + \frac{b_1}{Rr^2} - (D - \frac{3D}{r^4}) \cos 2\theta \\ \sigma_\theta &= 2S - \frac{b_1}{Rr^2} + (D - \frac{3D}{r^4}) \cos 2\theta \\ \tau_{r\theta} &= (D + \frac{3D}{r^4}) \sin 2\theta\end{aligned}\right\} \qquad (20)$$

and, since at the hole boundary r is unity,

$$\frac{A}{Rt} = \frac{(p+q) + 2\,\frac{b_1}{R} - 2(p-q)\cos 2\theta}{(1-\nu)(p+q) - 2(1+\nu)\,\frac{b_1}{R} + 2(1+\nu)(p-q)\cos 2\theta} \qquad (21)$$

Equations (20) and (21) clearly show a coupling through the term $\frac{b_1}{R}$. Actually, the presence of this term implies that several pairs of reinforcements and stress distributions are possible for a reinforced circular opening which do not alter the sum of principal stresses in the rest of the infinite plate under uniform biaxial tension at infinity. The possible values of p, q, and $\frac{b_1}{R}$, which do not permit the actual reinforcement distribution A/Rt to become negative, are given by the following inequalities:

$$\left.\begin{aligned}3 + 2\,\frac{b_1}{qR} > \frac{p}{q} > \frac{1}{3} - \frac{2}{3}\,\frac{b_1}{qR} \\ \frac{3+\nu}{1+3\nu} - 2\,\frac{1+\nu}{1+3\nu}\,\frac{b_1}{qR} > \frac{p}{q} > \frac{1+3\nu}{3+\nu} + 2\,\frac{1+\nu}{3+\nu}\,\frac{b_1}{qR}\end{aligned}\right\} \qquad (22)$$

These equations, if plotted on graph paper, will delimit an area which will define several permissible pairs of values for p/q and b_1/qR. It is obvious that, for p=q, Equation (21) reduces to the Mansfield solution [1]. At this point it should be noted that right from the beginning the problem did not call for a unique solution. The constraint of unaltered sum of principal stresses does not characterize the stress field as unique. As a result a number of possible solutions exist which satisfy the prescribed conditions. Any additional condition, such as regarding the stresses at any particular location, will immediately make the solution unique. The general procedure, as described, allows for some trade-off studies between the reinforcement distributions and the stresses. Of course, as pointed out by Bjorkman and Richards [2], other field conditions can also be prescribed as the beginning point.

To get an actual feeling for the solution, let

$$\frac{p}{q} = 1.5 \text{ and } \frac{b_1}{qR} = -0.5$$

Then

$$\frac{A}{Rt} = \frac{3 - 2\cos 2\theta}{(7-3\nu) + 2(1+\nu)\cos 2\theta}$$

which has a maximum value of $\frac{1}{1-\nu}$ at $\theta = \pi/2$ and a minimum value of $\frac{1}{9-\nu}$ at $\theta = 0$. This distribution obviously requires considerably less reinforcing material than the Mansfield case of all-around tension with $p/q = 1$ and a uniform $A/Rt = 1/1-\nu$.

ADDITIONAL EXAMPLES

Circular geometries are a special case in many physical problems and their solution is relatively simple. Therefore, it appears worthwhile to investigate some noncircular openings. Let an infinite plate under uniform biaxial tensions p and q along x- and y-axis, respectively, contain a reinforced opening given by

$$z = R(\zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3} + \frac{a_5}{\zeta^5}) \tag{23}$$

It is well-documented that a function such as Equation (23) can map a circular, elliptical, square, or rectangular opening in the z-plane onto a unit circle in the ζ-plane depending on the values of constants a_1, a_3, and a_5. Furthermore, if the orientation of the opening is symmetric with respect to the axes, then the constants a_1, a_3, and a_5 are real. Now with Equation (23) as the mapping function, the boundary condition (14) after usual processing yields

$$\left.\begin{aligned}
c_n &= d_n; \quad n \geq 0 \\
2c_0 &= Rb_1 + 2SR^2(1-a_1^2-3a_3^2-5a_5^2) \\
c_2 &= 2SR^2(a_1-a_1a_3-3a_3a_5) + DR^2 \\
c_4 &= 2SR^2(a_3-a_1a_5) \\
c_6 &= 2SR^2 a_5 \\
c_n &= 0; \quad n \geq 7
\end{aligned}\right\} \tag{24}$$

The function $\psi(\zeta)$ can be determined by integrating the conjugate of Equation (14). In fact, the coefficients are found to be:

$$\left.\begin{aligned}
b_{2n} &= 0; \quad n \geq 0 \\
b_3 &= a_1b_1 + \frac{c_2}{R} + 2SR(a_1+3a_1a_3+5a_3a_5) + DR(a_1^2+2a_3) \\
b_5 &= a_1b_3+3a_3b_1+\frac{c_4}{R}+2SR(3a_3+5a_1a_5)+4DR(a_1a_3+a_5) \\
b_7 &= a_1b_5+3a_3b_3+5a_5b_1+\frac{c_6}{R}+10SRa_5+3DR(2a_1a_5+a_3^2) \\
b_9 &= a_1b_7 + 3a_3b_5 + 5a_5b_3 + 8DRa_3a_5 \\
b_{11} &= a_1b_9 + 3a_3b_7 + 5a_5b_5 + 5DRa_5^2 \\
b_{13} &= a_1b_{11} + 3a_3b_9 + 5a_5b_7 \\
b_{2n+1} &= a_1b_{2n-1} + 3a_3b_{2n-3} + 5a_5b_{2n-5}; \quad n \geq 7
\end{aligned}\right\} \tag{25}$$

Now Equations (1A) and (5A) and the field condition can be combined to obtain a useful relation that will bypass the use of $\psi(\zeta)$; i.e., for computing A/t:

$$\frac{A}{t} = \frac{\frac{P}{t}\frac{ds}{d\beta}}{4S\frac{ds}{d\beta} - (1+\nu)\frac{P}{t}} \tag{26}$$

where

$$\frac{ds}{d\beta} = \frac{(z'\bar{z}')^{3/2}}{z'\bar{z}' + \frac{z'\bar{z}''}{2\sigma} + \frac{\sigma}{2}\bar{z}'z''}$$

is the radius of curvature of the opening boundary. The problem is thus solved. In fact, a direct substitution of the required quantities in Equation (26) leads to

$$\frac{A}{t} = \frac{P(\sigma)(z'\bar{z}')^{3/2}}{4S(z'\bar{z}')^2 - (1+\nu)P(\sigma)F(\sigma)} \tag{26A}$$

where

$$P(\sigma) = \sum_{n=0}^{3} c_{2n}\left(\sigma^{2n} + \frac{1}{\sigma^{2n}}\right)$$

and

$$F(\sigma) = z'\bar{z}' + \frac{1}{2\sigma}z'\bar{z}'' + \frac{\sigma}{2}\bar{z}'z'' \quad .$$

The mapping function z is given by Equation (23) and the constants c_n by Equation (24). The situation is again similar to that of the circular case. A/t is known within a constant b_1 which couples it with the stress field. Again a number of stress fields can be paired with appropriate reinforcements which will keep the sum of principal stresses unaltered over the entire field.

Using Equation (23) for a mapping function and assuming no reinforcement, the problem can be reduced to that of reference [2], which is to be expected. In contrast to reference [2], which determines the harmonic opening shape to suit an existing stress field, this paper proposes a method for reinforcing an opening of any desired shape in such a way that the reinforced opening behaves as a "harmonic hole."

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